# *More Recent Ideas about the AB Effect*

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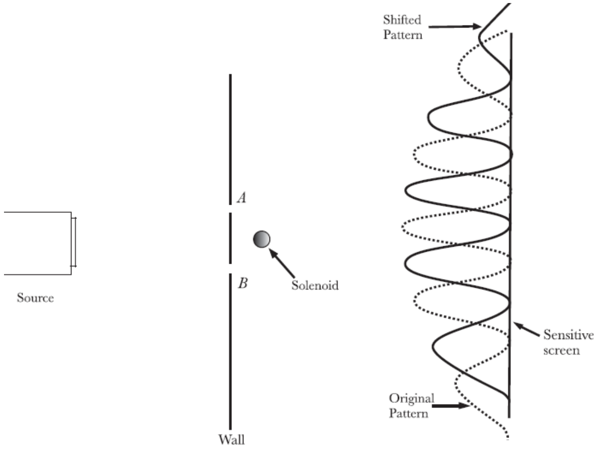
# The Aharonov-Bohm Effect

In the magnetic AB effect, a beam of charged matter forms an interference pattern on a screen after passing through a region containing a magnetic field confined to the interior of a solenoid.



The Aharonov-Bohm effect is usually thought of as a distinctively *quantum* phenomenon in which a classical electromagnetic field acts on quantum particles.
But this is only one (1) of four ways of modeling the effect:

- (2) one can model the beam of charged matter as a *classical* complex field;
- (3) one can model the *sources* of the classical field as well as the charged matter as quantum objects; or
- (4) one can model both charged matter and electromagnetism as quantized fields.

I'll look at each of these four ways, beginning with a purely classical model of the AB effect.

# A Purely Classical Model

- Here, charged matter is represented by the complex field $\phi(\mathbf{x}, t)$.
- The free field satisfies

$$\frac{\partial \phi}{\partial t} = \frac{i}{2m} \nabla^2 \phi \tag{1}$$

- When interacting with a classical magnetic field represented by the vector potential $\mathbf{A}(\mathbf{x}, t)$, this becomes

$$\frac{\partial \phi}{\partial t} = \frac{i}{2m} \left( \boldsymbol{\nabla} - ie\mathbf{A} \right)^2 \phi \tag{2}$$

The "local" gauge transformation

$$\phi \rightarrow \exp \left( ie\Lambda(\mathbf{x}, t) \right) \phi, \qquad \mathbf{A} \rightarrow \mathbf{A} + \boldsymbol{\nabla} \Lambda \tag{3}$$

is a symmetry of solutions of equation (2).

- This reproduces the magnetic AB fringe shift if the matter intensity at the screen is proportional to $|\phi(\mathbf{x})|^2$:
  in the steady state, the shift $\Delta$ depends only on the (gauge invariant) integral of $\mathbf{A}$ around a loop $\gamma$ encircling the solenoid, $\oint_\gamma \mathbf{A}(\mathbf{x}).d\mathbf{x}$ (i.e. the magnetic flux through the solenoid).

- The striking thing is that the effect occurs even though the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ is then zero everywhere in the region through which the charged matter passes, no matter what constant current is passing through the solenoid.

- This purely classical model of the AB effect is not usually taken seriously. The effect occurs in a low-energy, non-relativistic regime in which charged matter is modeled very well as composed of charged particles that are neither created nor destroyed. In a purely classical model these particles don't display interference, and are influenced by magnetic fields only according to the Lorentz force law $\mathbf{F} = e\mathbf{v} \times \mathbf{B}$.

# The "Usual" Quantum Model

- The purely classical model is formally identical to the usual way of modeling the AB effect in terms of the influence of an external classical magnetic field on non-relativistic quantum particles: putting the wave-function $\psi(\mathbf{x}, t)$ in place of $\phi(\mathbf{x}, t)$ and choosing units in which $\hbar = 1$ turns (2) into the Schrödinger equation with an external classical magnetic field represented by **A**.

Here are three ways of understanding classical magnetism in the AB effect, whether this is taken to act on a charged classical field $\phi(\mathbf{x}, t)$ or the quantum wave-function $\psi(\mathbf{x}, t)$ of charged particles.

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1. The only real field is $\mathbf{B}$. $\mathbf{B}$ acts non-locally on $\phi$ ($\psi$).
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3. The really fundamental field is $h_{\mathbf{A}}(\gamma) \equiv \exp(ie\oint_{\gamma}\mathbf{A}(\mathbf{x}).d\mathbf{x})$—the gauge-invariant magnetic holonomy of smooth closed curves $\gamma$. $h_{\mathbf{A}}(\gamma)$ represents magnetism completely and non-redundantly: $\mathbf{B}(\mathbf{x})$ arises as the limit of $h_{\mathbf{A}}(\gamma)$ for loops $\gamma$ converging on $\mathbf{x}$.

In my 2007 *Gauging What's Real* I defended this third way of understanding the ontology of classical magnetism, extended first to classical electromagnetism *via* $h_A(\gamma) \equiv \exp\left(-ie\oint_\gamma A_\mu(x).dx^\mu\right)$ for space-time curves $\gamma$ with 4-vector (or 1-form) potential $A_\mu$, and then again to (even non-Abelian) pure Yang-Mills fields *via* $h_{\vec{A}}(\gamma) \equiv \mathcal{P}\exp\left(-ig\oint_\gamma \vec{A}_\mu(x).dx^\mu\right)$ where $\vec{A}_\mu$ is a Lorentz 4-vector and also a vector in (a representation of) the internal structure group of the theory (e.g. $SU(3)$ for a pure color classical gauge field).
(The need to path-order the exponential arises in a non-Abelian theory for which $\left[\vec{A}_\mu(x), \vec{A}_\mu(x+dx)\right] \neq 0$.)

- On this view, a classical pure Yang-Mills gauge theory (including Maxwellian electromagnetism) has a *non-separable* ontology: the qualitative intrinsic properties it represents in space(-time) region $R_1 \cup R_2$ are not always determined by (supervenient upon) those it represents in regions $R_1$, $R_2$.

# The Sources of **B** as Quantum Objects

Lev Vaidman has recently argued for a local, *separable* account of the AB effect in which the only real field involved is the classical electromagnetic field (**E**,**B**).

> *"We might change our understanding of the nature of physical interactions back to that time before the AB effect was discovered. The quantum wave function changes due to local actions of fields."*

Vaidman's idea is to include sources of the (classical) EM field in a model. The effect is supposed to arise in a three stage process.

1. The motion of the charged particles (electrons) produces an EM field that acts locally on each component of the *source's* wave function.
2. This directly affects the relative phase of components of the *total* wave function of source+electrons.
3. As the electrons separate from the source, this relative phase is transferred to components of the electrons' wave function as the AB phase difference.

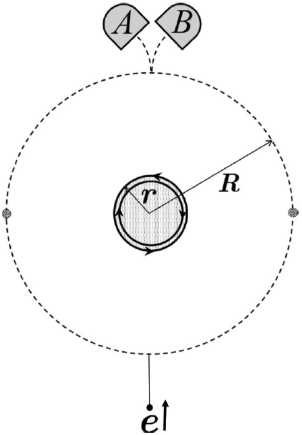



FIG. 4. The magnetic AB effect. The electron wave packet coming directly toward the solenoid splits into a superposition of two wave packets which encircle the solenoid from two sides and come out almost in the same direction, interfering toward detectors $A$ and $B$.

The source here is modeled by two counter-rotating very long cylinders with surface velocities $\pm v$, and equal and opposite charges $\pm Q$.

As an electron's wave packet passes by on either side of the cylinders, its electric field on each branch changes the magnetic flux through the cylinders, and this induces an electromotive force that changes their angular velocities.

Vaidman treats the source cylinders as a quantum object and assigns it a wave-function $|\Psi\rangle_S$, while the two components of the electron wave-function are written $|L\rangle_e$, $|R\rangle_e$. He argues that the electron's effect on the source produces an AB phase difference $\varphi_{AB}$ between two components of the *source* wave-function, $|\Psi_L\rangle_S$ and $|\Psi_R\rangle_S$ that transfers to the same phase difference between $|L\rangle_e$and $|R\rangle_e$. He writes the evolving total wave-function as the electron passes as

$$\frac{1}{\sqrt{2}}\left(|L\rangle_e |\Psi_L\rangle_S + |R\rangle_e |\Psi_R\rangle_S\right) \quad (4)$$

and claims that this becomes separable before the electron reaches the detectors

$$\frac{1}{\sqrt{2}} |\Psi\rangle_S \left(|L\rangle_e + e^{i\varphi_{AB}} |R\rangle_e\right). \quad (5)$$

# Comments on Vaidman

1. It is important to the argument that entanglement disappears before the detectors, otherwise decoherence would wash out interference. But Vaidman argues that prior entanglement is *not* important—a pure product state throughout would suffice.
2. Vaidman did not answer Aharonov's objection based on Tonomura's experiment that found the magnetic AB effect even with a source completely shielded to prevent it from generating external magnetic fields. **Claim**: This also prevents the electrons from changing the flux through the source, ruling out Vaidman's explanation of the effect.
3. As he acknowledges, Vaidman's argument cannot be made rigorous by exact quantum calculations since in standard quantum mechanics such calculations require *potentials*.
4. Vaidman says the phase is gradually acquired by the source of the electromagnetic potential. But the phase of a wave-function at a point here is defined only *after* one chooses a gauge for **A**: different choices redistribute phase changes between $|\Psi_L\rangle_S$ and $|\Psi_R\rangle_S$. The electrons can have no local, gauge-independent effect on the source.

# Wallace's New View

- I used to favor a nonseparable acount of classical gauge fields when I held a non-separable interpretation of quantum mechanics. Putting this together with non-separable classical electromagnetism promised a *local* but non-separable account of the AB effect in non-relativistic quantum mechanics.
- But David Wallace has recently proposed a local, *separable* account of the purely *classical* AB effect and considered extending it by *quantizing* these classical fields.
- The purely classical magnetic AB effect classical fields satisfy

$$\frac{\partial \phi}{\partial t} = \frac{i}{2m} \left( \boldsymbol{\nabla} - ie\mathbf{A} \right)^2 \phi \tag{2}$$

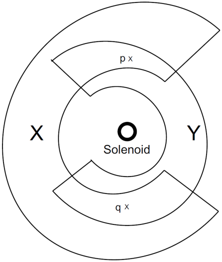
Neither of these fields is invariant under the local gauge transformations (3) $\phi \rightarrow \exp\left(ie\Lambda\right)\phi$, $\mathbf{A} \rightarrow \mathbf{A} + \boldsymbol{\nabla}\Lambda$. But we can find such gauge invariant quantities. Write $\phi(\mathbf{x}, t) = \rho(\mathbf{x}, t) \exp ie\theta(\mathbf{x}, t)$ with $\rho$ a non-negative real field. $\rho$ itself is locally gauge invariant: so is $\boldsymbol{\nabla}\theta - \mathbf{A}$. Write these as $\rho$, $\mathcal{D}\theta$.

- $\rho$ and $\mathcal{D}\theta$ are also *locally defined* quantities. Wallace argues they permit a local, separable account of the magnetic AB effect.
  He shows that equation (2) may be rewritten as a set of gauge invariant equations

$$\begin{aligned} \left[\nabla^2 - (\mathcal{D}\theta)^2\right]\rho &= 0 \\ 2\mathcal{D}\theta \cdot \boldsymbol{\nabla}\rho + (\boldsymbol{\nabla}.\mathcal{D}\theta)\,\rho &= 2m\dot{\rho} \\ \boldsymbol{\nabla}\times\mathcal{D}\theta &= 0 \end{aligned} \tag{6}$$

- This suggests a *fourth* way to understand a classical Yang-Mills gauge theory: it represents a *single* field with amplitude $|\phi| = \rho$ and (generalized) phase $\mathcal{D}\theta$.
  In this view, classical matter and electromagnetism are *not* distinct entities, each with its own representation, but different gauge-dependent aspects of a unified entity whose ontology is better represented gauge-independently in terms of "components" $\rho, \mathcal{D}\theta$ that can be decomposed into $\phi, \mathbf{A}$ only in an arbitrary gauge-dependent way. So asking for the ontology of a pure Yang-Mills gauge field like electromagnetism was asking the wrong question!

But shadows of non-separability remain. Consider overlapping spatial regions $X$, $Y$ outside the solenoid whose union $X \cup Y$ contains closed paths encircling it.




- There is a path $\alpha$ from $p$ to $q$ in $X$ and another path $\beta$ from $p$ to $q$ in $Y$: let $\gamma$ be the closed path $\alpha^{-1} \circ \beta$ at $p$. $\int_\alpha \mathcal{D}\phi$, $\int_\beta \mathcal{D}\phi$ are each gauge invariant, and $\oint_\gamma \mathcal{D}\phi = e\oint_\gamma \mathbf{A}(\mathbf{x}).d\mathbf{x}$. The flux through the solenoid determines both $e\oint_\gamma \mathbf{A}(\mathbf{x}).d\mathbf{x}$ and the interference pattern: it does *not* determine $\int_\alpha \mathcal{D}\phi$, $\int_\beta \mathcal{D}\phi$ individually, but only their difference. If observation of the interference pattern does not permit measurement of these phase-differences along open paths, what does?

- This question highlights the artificiality of treating the AB effect in terms of purely classical fields. Of course we can't measure $\int_\alpha \mathcal{D}\phi$, $\int_\beta \mathcal{D}\phi$ individually, because charged matter in our world is not adequately represented by a classical field!
- If we insist on the usual way of treating the AB effect in non-relativistic quantum mechanics, in terms of the action of an external classical field on charged quantum particles, then Wallace's view is not applicable.
- But what he had in the back of his mind all along was not way #2, but way #4—a fully quantum theoretical treatment in terms of interacting *quantum* fields. How well does Wallace's view work there?

# The AB Effect with Interacting Quantum Fields

His idea is to treat the AB effect as a local interaction between two "entities": the ground state of the quantized field surrounding the solenoid and charged particles, considered as excitations of the field forming incoming wave-packet states. He accepts that

1. There is no completely precise separation between these "entities".
2. Single particle wave-packet excitations are only precisely defined for a non-interacting field.

But he notes that this separation is quite distinct from a (gauge-dependent!) separation of the interacting quantum fields into a fermion field $\hat{\psi}$ and a boson field $\hat{A}_\mu$.
One can identify the local "mechanism" of the AB effect as vacuum polarization of the region around the solenoid corresponding to the expectation value of the covariant derivative of the phase

$$\left\langle \hat{\rho}(r)^{-1}\widehat{\mathcal{D}_\theta \psi}(r) \right\rangle = \frac{(e\Phi)_{\mathrm{mod}\, 2\pi}}{2\pi r} \tag{7}$$

# Some Initial Reactions

According to Wallace, if we model the magnetic AB effect in terms of a system of interacting quantum fields (a fermion field and the quantized Maxwell field) then to a very high degree of accuracy we can regard the solenoid region and the wave packet as separate entities which interact with each other.

- Whatever they are, these "entities" are not to be found in the fundamental ontology of any physical theory. They are, at best, *functional realizations* of such an ontology—dynamic patterns of an emergent ontology. What can said about a more fundamental ontology from which these "entities" emerge?
- As stated, they seem to be *properties*, not entities: each is rather a quantum state of something—but what? A physical system represented by interacting quantum fields?
- If the wave packet state is a perturbation of the state of the region around the solenoid, then how could these states be said to be separate, or to *interact*?

- What all these questions are pointing to is the vexed question of the ontology of a quantum field theory: what is a quantum field theory really *about*—particles, fields, loops or something else entirely?
- My currently favored view of quantum theory has a simple, though perhaps disappointing, answer: *a quantum field theory has no ontology of its own*!
- In this view, the function of a model of quantum field theory is not to describe or represent some novel quantum structure but to offer its user wise advice on the content and credibility of claims attributing values of magnitudes to non-quantum physical systems, including classical particles and fields.
- The AB effect occurs only if the charged matter is not decohered by its environment before the screen, so no claim about it there has rich enough content to be worth entertaining. Decoherence at the screen endows certain claims about physical systems involved in detection of the beam with a rich enough content to license application of the Born rule to a quantum state of those systems.

- A wise user of quantum theory will adjust credences to match the resulting Born probabilities. In this way the user will not be surprised by the AB effect: knowing also on what physical conditions it depends (including the current flowing through the solenoid as well as how the beam was prepared) the agent can use a model of quantum field theory to *explain* the effect. But the explanation will not involve any ontological claims about properties of charged matter in the interferometer or the physical condition of the region surrounding the solenoid.
- *Contra* Wallace, quantum field theory neither provides nor underwrites a local, causal account of the AB effect in terms of interacting entities.